Social ranking rules for incomplete power relations

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Power Relation (Binary relation)

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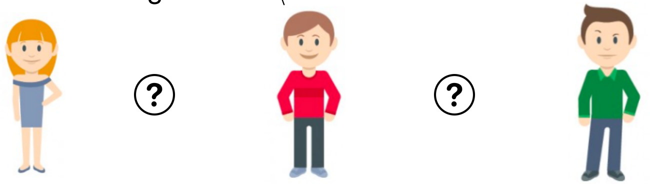
Social ranking solution\ Axiomatization

Linear Order



Objective

Input :

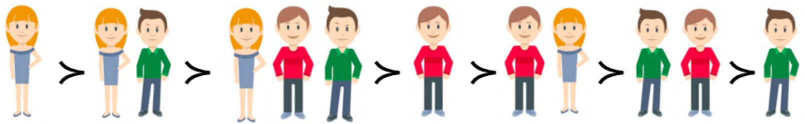
- A set of individuals : $N = \{1, \ldots, n\}$
- A *power relation* $\succeq$ on 2^N :
  $S \succeq T$: The "team" S performs at least as good as T.

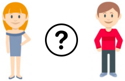
  We suppose $\succeq \in \mathcal{B}(2^N)$, set of all binary relations.

Output :

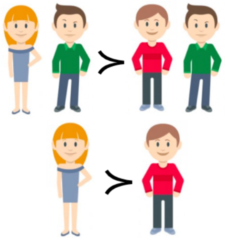
- A *solution* $R^{\succeq}$ ($I^{\succeq}$ the symmetric part, $P^{\succeq}$ the strict part), associates to every power relation ($\succeq$) a ranking (linear order) over the set of individuals.

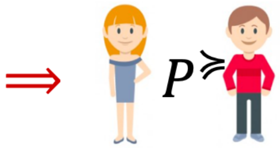
Pair-wise Ceteris-Paribus majority rule





Informative part : CP-comparisons





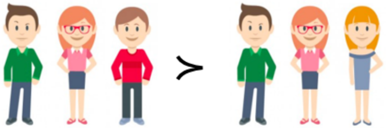
Interpretation : Electoral system

Ceteris Paribus principle transforms the problem to a kind of electoral system with two differences :

- Voters are coalitions : the interaction among the members who form the coalitions (voters) are important,
- Each coalition can do compare individuals that are not in the coalition. Thus one individual can be a part of voter and also be a candidate at the same time.

Coalitions as Voter (Issue 1)

- What interaction between individuals show ?



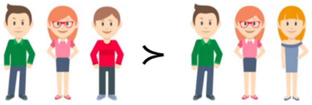
For instance in some context the related questions may be :

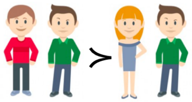
- Do the members reach an agreement in democratic way ?
- Or is there one who imposes his or her opinion ?

Coalitions as voters Issue 2

- What bout the size of coalitions ?

Preferences made by which coalition worth more ?





Issue 3

Coalitions have different sets of individuals to compare :

- Let's set N is :

- Coalition  can compares individuals  .
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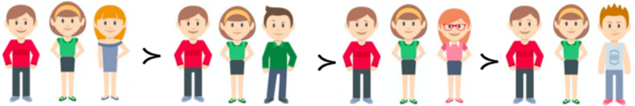
Weighted version of CP-majority rule

By all these considerations :



Preference of  to  accompanied by a weight depends on :

- Other members get compared by the coalition,
- Worth of individuals in the coalition and their interaction, Set.png 

Ranking more than two individuals

$$135 \succ 235 \succ 345 \succ 25 \succ 15$$

The goal is to compare $1, 2, 3, 4, 5$

- $\succeq_S = \{(i, j) | i \cup S \succeq j \cup S \ s.t \ i, j \in N, i, j \notin S, i \neq j\}$
- $\succeq_{\{3,5\}} = \{(1, 2), (1, 4), (2, 4)\}$, $\succeq_{\{5\}} = \{(2, 1)\}$
- We refer to space of all linear orders on the set $N = \{1, 2, 3, 4, 5\}$
- We choose the one which is closer to the provided preferences by information sets :

$$F_w(\succeq) = \underset{R \in \mathcal{L}(N)}{\text{argmax}} \sum_{S \in 2^N} w(S, \succeq_S) \cdot |R \cap \succeq_S|$$

Ranking more than two individuals (Example)

$$135 \succ 235 \succ 345 \succ 25 \succ 15$$

- $\succeq_{\{3,5\}} = \{(1,2),(1,4),(2,4)\}$, $\succeq_{\{5\}} = \{(2,1)\}$
- $F_w(\succeq) = \underset{R \in \mathcal{L}(N)}{\text{argmax}}[w(\{3,5\},\succeq_{\{3,5\}}) \cdot |R \cap \{(1,2),(1,4),(2,4)\}|$
  $+w(\{2,1\},\succeq_{\{2,1\}}) \cdot |R \cap \{(2,1)\}|]$
- Suppose $w(\{3,5\},\succeq_{\{3,5\}}) = 2, w(\{5\},\succeq_{\{5\}}) = 1$ Then :
  $\{(1,2),(1,4),(2,4),(1,5),(2,5)\} \subset R \subset F_w(\succeq)$
  $\{(1,2),(1,4),(2,4),(5,1),(5,2)\} \subset R' \subset F_w(\succeq)$

Problem definition

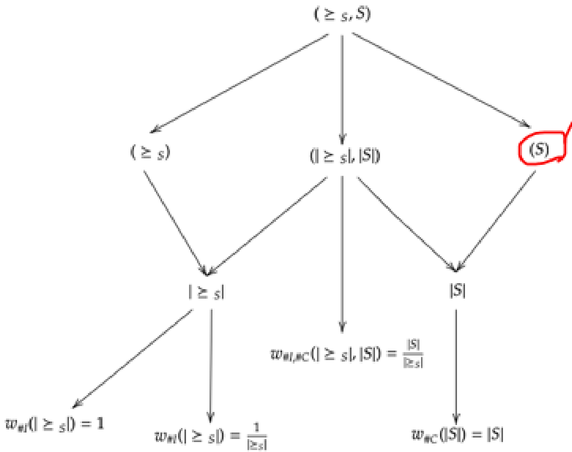
Input :

- A set N of individuals,
- The informative part of a power relation $\succeq \in \mathcal{B}(2^N)$: $\{\succeq_S, S \in 2^N\}$,
- A defined weight function w,

Output :

- A set of linear orders on set N of individuals who are more closer to the preferences in the informative part.

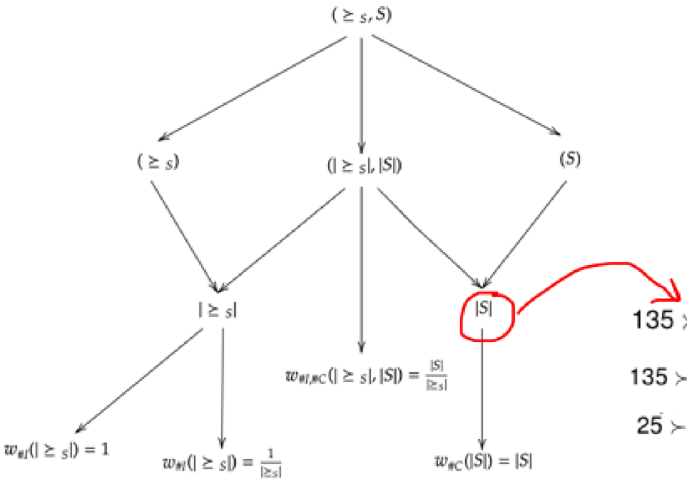
Tree Structure



$135 \succ 235 \succ 345 \succ 25 \succ 15$

$135 \succ 235$ $\quad w(\{3,5\}) = ?$

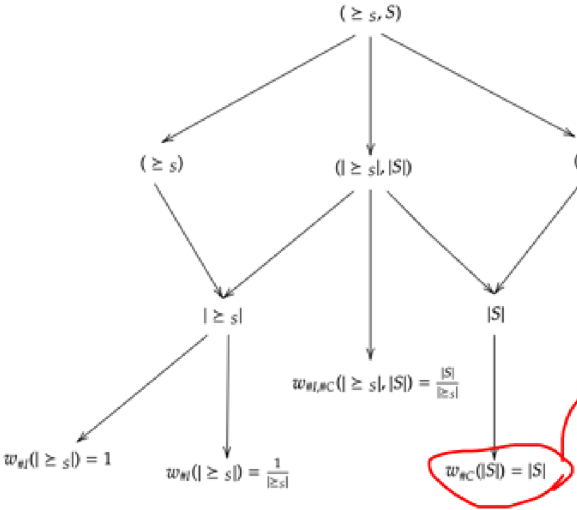
$25 \succ 15$ $\quad w(\{5\}) = ?$



$135 \succ 235 \succ 345 \succ 25 \succ 15$

$135 \succ 235 \qquad w(|\{3,5\}|) = ?$

$25 \succ 15 \qquad w(|\{5\}|) = ?$



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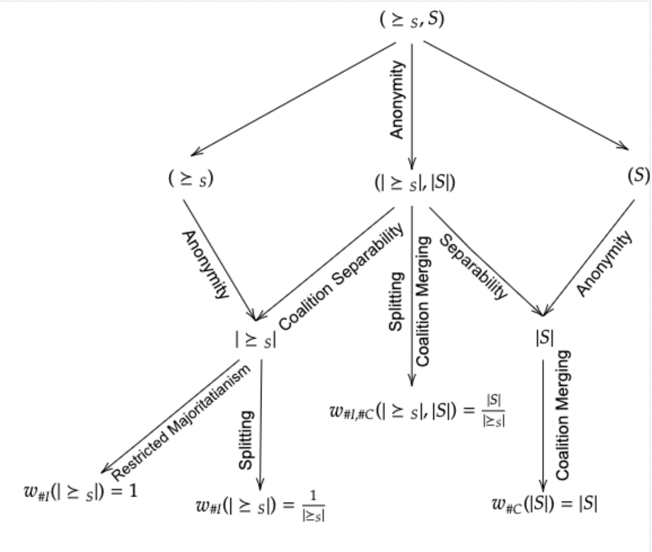

Splitting axiom(example)

- $N = \{1, 2, 3, 4, 5\}$
- $\succeq_{\{1\}} = \{(3, 4)\}, \succeq_{\{2\}} = \{4, 5\}$
- $\sqsupseteq_{\{1\}} = \{(3, 4), (4.5)\}, \sqsupseteq_{\{2\}} = \{(3, 4), (4, 5)\},$
- If F_w satisfies Splitting, it holds that $F_w(\succeq) = F_w(\sqsupseteq)$.

Splitting (Formal definition)

Definition (Splitting axiom)

A ranking rule F satisfies splitting if and only if for any two given power relations $\succeq, \sqsupseteq \in \mathcal{B}(2^N)$ and a set of individuals $\{i_1, j_1, i_2, j_2, ..., i_\ell, j_\ell\} \subset N$, $\ell \in \mathbb{N}$ if the two power relations are identical except for a set of coalitions of the same size $\{S_1, ..., S_\ell\}$ such that $i_1, j_1, i_2, j_2, ..., i_\ell, j_\ell \notin S_1, ..., S_\ell$ and $\{i_1 j_1\} =_{\succeq} S_1, \{i_2 j_2\} =_{\succeq} S_2, ..., \{i_\ell j_\ell\} =_{\succeq} S_\ell$ while $\{i_1 j_1, i_2 j_2, ..., i_\ell j_\ell\} =_{\sqsupseteq} S_1 = ... =_{\sqsupseteq} S_\ell$ then it holds that $F(\succeq) = F(\sqsupseteq)$.

Theorem

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The only weighted ranking rule of the family $\mathcal{F}_{w_{\#I}}$ that satisfies splitting is $F^{p}_{w_{\#I}}$.

- $F_{w_{\#I}}(\succeq) = \underset{R \in \mathcal{L}(N)}{\text{argmax}} \sum_{S \in 2^N} w_{\#I}(|\succeq_S|) \cdot |R \cap \succeq_S|$
- $F^{p}_{w_{\#I}}(\succeq) = \underset{R \in \mathcal{L}(N)}{\text{argmax}} \sum_{S \in 2^N} \frac{1}{|\succeq_S|} \cdot |R \cap \succeq_S|$